Name————————————————, ID ————————————

MTH 330, Review for Final Exam, Fall 2014

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QUESTION 1. (i) To tile a floor, we may use pieces of a regular 12-gon with :

a) pieces of regular 3-gon and pieces of regular 6-gon b) pieces of regular 8-gon c) pieces of regular 4-gon d) pieces of regular regular 6-gon and pieces of regular 6-gon.

(ii) To tile a floor, we may use pieces of regular 4-gon with:

a) pieces of regular 12-gon and pieces of regular 3-gon b) pieces of regular 8-gon and pieces of regular 3-gon. c) pieces of regular 3-gon. d) (a) or (c).

(iii) To a tile a floor, we may use pieces of regular 8-gon with:

a) pieces of regular 3-gon b) pieces of regular 4-gon c) pieces of regular 6-gon d) (a) or (b)

(iv) Let C be a circle of radius 4 centered at O, and A is a point inside C such that $| OA |= 2$. Then $| AInv(A) |$ =

a) 8 b)6 c) 4 d) 10

(v) Let C be a circle centered at A with radius 6 and D is another circle with radius 2 centered at B such that D is passing through A. Then the inversion of D with respect to C is :

a) a line that is perpendicular to the line AB at a point F such that $|AF| = 9$ b) a line that is perpendicular to the line AB at a point F such that $|AF| = 3$ c) a circle with radius 3 passing through A d)a circle with radius 4 passing through A .

(vi) Let C be a circle centered at O. Given A, B are points such that O, A, B lie on the same line. Given $| OA |<| OB |$. Then

a) $| Inv(A)Inv(B) |=| AB |$ b) $| OInv(A) |<| OInv(B) |$ c) $| OInv(B) |<| OInv(A) |$ d) We can not tell

(vii) The measurement of each vertex-angle of a regular 20-gon is

a) 144 (b) 162 c) 18 36

(viii) One of the following is constructible by unmarked ruler and a compass:

a) regular 26-gon b) regular 40-gon c) regular 34-gon d) regular 54-gon

(ix) Given C is a circle centered at O and with radius 6 cm. Let A be a point such that $| OA |= 3$. Let $Inv(A)$ be the inversion of A with respect to C. Then $| OInv(A) |$ =

a) 2 b)12 c) 9 d) 4.5

(x) Using unmarked ruler and a compass:

a) We can construct a 48 degree angle. b) We can construct a 10 degree angle. c) We can construct a 55 degree angle. None of the previous is true.

(xi) If an angle α is constructible, then the angle $\alpha/16$ is constructible.

a) True b) False

(xii) Let C be a circle centered at O and with radius 3. Given A is a point such that $\mid OA \mid= 1$ and D is a circle orthogonal to C and passing through A. Then one of the following values is a possibility for the radius of D:

a)3 b)5 c) 3.5 d) 2

(xiii) Let H be the horizon circle (the model for non-Euclidean) with radius 4 and centered at O. Let A be a point in H such that $\mid OA \mid= 3$. Then the non-Euclidean distance between O and A is :

a) ln(3) b) ln(7) c) ln(9) = 2ln(3) d) ln(4)

(xiv) In non-Euclidean (hyperbolic) geometry, if a, b are two points, then

a) There are infinitely many lines pass through a and b b) There is exactly one circle passes through a and b c) There is exactly one line passes through a but not through b d) There is exactly one line passes through a and b.

(xv) In non-Euclidean Geometry, the sum of all interior angles of a regular 4-gon is

a) 180 b) less than or equal to 180 c) 360 d) less than 360

(xvi) Let C be a circle with radius 4 and centered at O. Let Q be a point on C. Draw a circle call it D centered at Q with radius 4 again (note that D passes through O). The two circles intersect in two points, say A and B. Now choose a point say Z on D such that the line segment OZ is a diameter of D. Now the line segment AB intersects the diameter OZ in a point say M (note that AB is perpendicular to OZ). The inversion of M with respect to the circle C is

a) the point Z b) a point outside the circle D c) a point outside C but inside D and not on D d) is the mid point of the line segment QZ.

(xvii) In the previous question, the length of AZ is

a) 4 b) $4\sqrt{3}$ c) 6 d) $2\sqrt{3}$

(xviii) The length of AQ in question XIII is

a) 2 b) $\sqrt{2}$ c) $2\sqrt{3}$ d)$4\sqrt{3}$

(xix) Let K be the mid-point of the line segment OM as in question XIII. The inversion of K with respect to C is

a) a point inside D but outside C b) the mid-point meter OZ c) the mid-point of QZ d) a point outside D but on the line extension of OZ

QUESTION 2. Fill in the blank

(i) Let C be a circle of radius 3 centered at O, A and B are points such that $|AO| = |BO| = 1$ and the angle AOB is a right angle at O. The radius of the circle that passes through $A, Inv(B)$ and orthogonal to C is ———————————————————

(ii) Let C be a circle of radius 3 centered at O, D is a circle centered at F such that $|FO| = 1$ and of radius 4. Then the Inv(D) with respect to C is a circle centered at B where $|AB| =$ ————————————- and it has radius ———————————————————

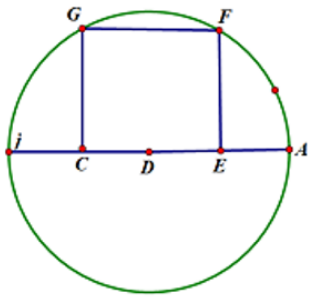
(iii) Let C be a circle of radius 3 centered at O, D is a circle centered at F such that $|FO| = 5$ and of radius 5. The Inv(D) with respect to C is ————- that is perpendicular to the line OF and intersects OF at a point W such that $|OW| =$ ———————————————————-

(iv) Let C be a circle with radius 5 and centered at $(0, 0)$. the inversion of the point $(6, 8)$ with respect to C is the point ———————— and the inversion of the point $(-2, 1)$ is the point ————————————-

(v) Given a line segment AB of length x. The following steps will be used to construct a line segment of length $\sqrt{5}x$ ————————————- and the following steps are used to construct a line segment of length $\frac{4x}{\sqrt{5}}$. In addition, if a line segment of length y is given, the following steps are used to construct a line segment of length $\sqrt{2xy}$ ———————————— ———————————————————————.

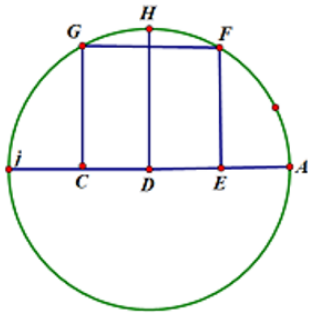
If $x > 1$ and a line segment of length one is given, then the following steps are used to construct a line segment of length z such that $xz = y$. If $x > 8$, then the following steps are used in order to construct the golden cut on AB .....

Only unmarked ruler and a compass are allowed. See diagram. D is the center of the circle (D is the midpoint of IA).
CLEARLY STATE the steps in order to construct the square FECG



QUESTION 4. Make sure that your solution is readable.

Only unmarked ruler and a compass are allowed. See diagram. D is the center of the circle (D is the midpoint of IA).
CLEARLY STATE the steps in order to construct the rectangle FECG such that |FE| = 1.5|CE| + 0.5|DH|



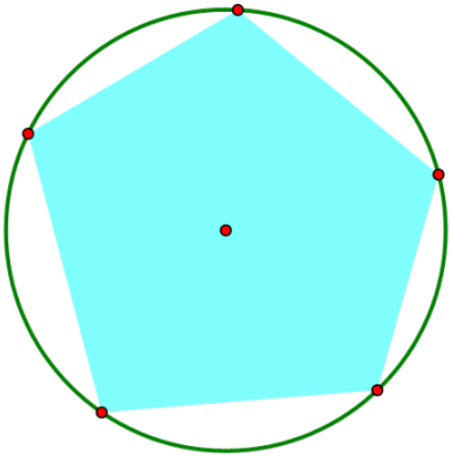
QUESTION 5. Make sure that your solution is readable.

Consider the line segment CD. Given E is the golden cut point of CD. Just do one step in order to locate the golden cut point of CE.

QUESTION 6. Make sure that your solution is readable.

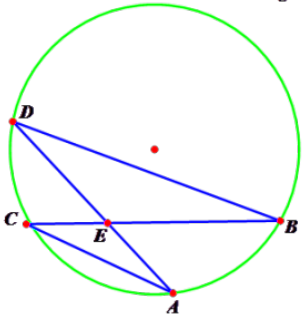
Consider the line segment AB where D is the golden cut point of AB. Now, tell me how will you construct 72 degree angle?

You may want to use the above to construct regular 5-gon as below. Tell me how?



QUESTION 7. Make sure that your solution is readable.

Consider the diagram below. Given degree measure of the arc (clock wise) DB = 200 degrees, the angle DEB = 115 degrees, the degree measure of the arc CD = 60 degrees. Find the degree measure of the arc BA, find the angles DBC, BDA, BCA, DAC



QUESTION 8. Make your solution readable.

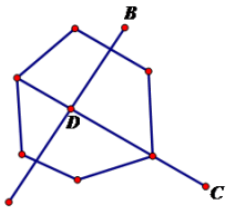
Given the line segment AD. State clearly the steps you will do in order to split the line segment AD into 3 parts such that |BC| = 2.5|AB| and |CD| = 3/4|AB|

QUESTION 9. Make sure that your solution is readable

The below is regular 6-gon. How many reflections does it have?

What is the angle of rotation for R1? for R3? Note that D is the center point. Find (R3 O B), see B below.

Find (C O R_2), see C below.



QUESTION 10. Using a compass an unmarked ruler only:
Can we construct a 40 degree angle? explain?

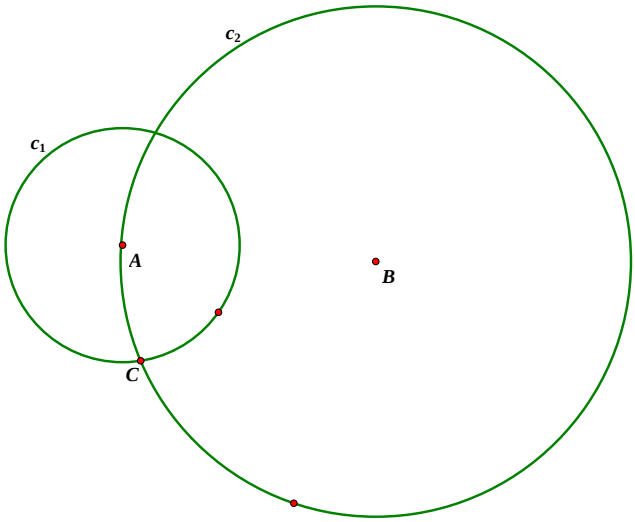
Can we construct a regular 26-gon?explain.

Can we construct a 75-degree angle? explain.

QUESTION 11. Can we tile a floor using regular 6-gon and regular 4-gon and regular 3-gon?

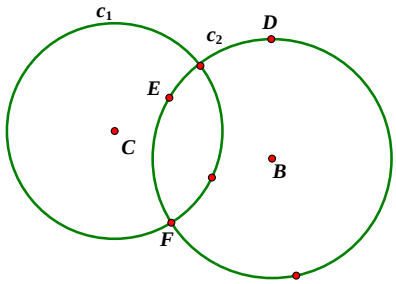
We can tile a floor with regular 12-gon with other regular n-gon? Find all possible values of n?

QUESTION 12. Make sure that your solution is readable.

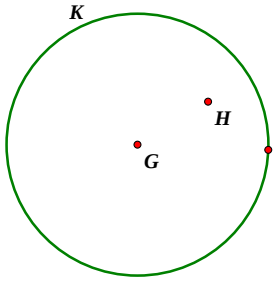


C1 is centered at A. C2 is centered at B. Construct the exact inversion of the ARC, AC (Clockwise) of C2 with respect to C1.

QUESTION 13. Make sure that your solution is readable.

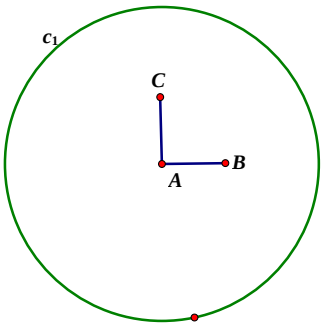


C1 is centered at C. C2 is centered at B. The inversion of the point E with respect to C1 is the point D. Construct the exact inversion of the ARC , EF (Clockwise), of C2 with respect to C1. Assume the radius of C1 is equal to the radius of C2 = 3. Let L be the inversion of the point B with resprct to C1. Find the exact length of of the line segment CL, i.e., find |CL|.

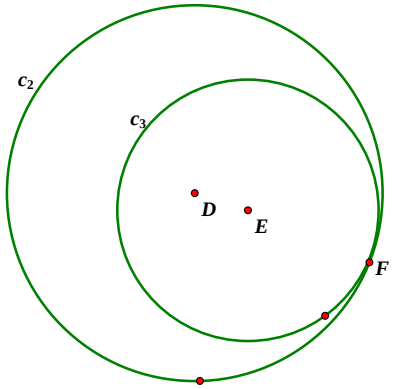


Given K centered at G and with radius 6cm. Given |GH| = 3. Let L be the inversion of H with respect to K. State the steps needed in order to construct a circle , say W, passes through H and L such that W is of radius > 6.

QUESTION 14. Make sure that your solution is readable. (**NOTE THAT AB is perpendicular to AC**)

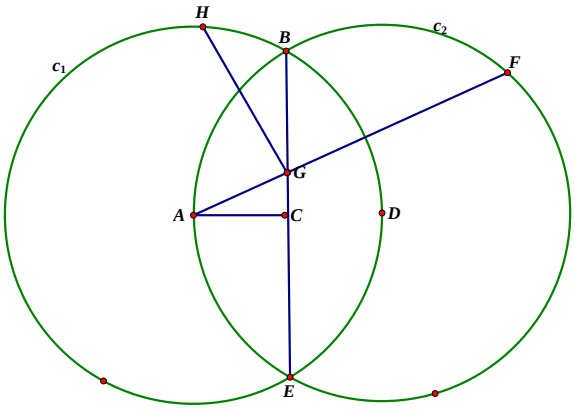


C1 centered at A and it has radius 4. |AB| = 1cm and |AC| = 2cm. Find the exact radius of the circle that passes through C, B and perpendicular to C1.

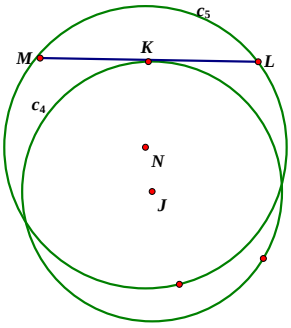


C2 centered at D and of radius 3. C3 is centered at E and of radius 2 and it intersects C2 at the point F. Given D, E, and F lie on the same line. Let C be the inversion of C3 with respect to C2. Find the exact location of the center of C. Find the exact radius of C.

QUESTION 15. Make sure that your solution is readable.



C1 Centered at A with radius 4. C2 centered at D. Given AC is perpendicular to BE, and |GC| = 1cm. Find the length of |AF|. If HG is perpendicular to AF, find |HG| and then find the length of the linesegment FH.



C4 centered at J. C5 centered at N. |LJ| = |MJ|. Roughly, construct the inversion of the line segment LM with respect to C4, and construct the inversion of the ARC LM of C5 with respect to C4

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